Hydraulics of pipelines
Application of Bernoulli equation **BE**
continuity equation **CE**

**BE 1 - 2**

\[ h_1 + \frac{p_1}{\rho g} + \frac{\alpha v_1^2}{2g} = h_2 + \frac{p_2}{\rho g} + \frac{\alpha v_2^2}{2g} + Z \]

\( Z \) – loss head (losses):
- **friction losses** \( Z_t \) (in distance \( L \))
- **local losses** \( Z_m \)

\[ Z = Z_t + Z_m \]

**CE 1 - 2**

\[ Q_1 = Q_2 \Rightarrow v_1 \cdot S_1 = v_2 \cdot S_2 \]
FRICTION LOSSES

uniform flow $\emptyset D = \text{const}, \nu = \text{const}$

$i_E = \text{hydraulic slope (friction slope)} = \text{slope of EL}$

$Z_t = Z_t (\nu, D, L, \text{character of walls of pipeline})$
Equilibrium of forces in elementary volume EV

\[(p + dp) \cdot S - p \cdot S = \tau_0 \cdot O \cdot ds\]
\[\frac{dp}{ds} = \tau_0 \cdot \frac{O}{S}\]

for circle pipeline $\varnothing D$: \[O = \pi \cdot D, \quad S = \frac{\pi \cdot D^2}{4} \Rightarrow \frac{O}{S} = \frac{\pi \cdot D}{\pi \cdot D^2} = \frac{4}{D}\]

\[\frac{dp}{ds} = \frac{\tau_0 \cdot 4}{D}\]
Expression of $\tau_0$ under turbulent flow

friction coefficient:

$$f_f = \frac{\text{shear stress } \tau_0}{\text{kinetic energy in unit volume}} = \frac{\tau_0}{1/2 \cdot \rho \cdot v^2}$$

$$\lambda = 4 \cdot f_f = \frac{8 \cdot \tau_0}{\rho \cdot v^2} \Rightarrow \tau_0 = \frac{\lambda}{8} \cdot \rho \cdot v^2$$

coefficient of friction loss $\lambda = f (\text{Re}, \Delta/D)$

Darcy-Weisbach equation

$$i = \frac{\text{d}p}{\text{d}s} = \frac{\tau_0 \cdot 4}{D} = \frac{\lambda}{8} \cdot \rho \cdot v^2 \cdot \frac{4}{D} = \frac{\lambda}{D} \cdot \frac{1}{2} \cdot \rho \cdot v^2$$

uniform flow: $PL \parallel EL \Rightarrow \frac{\Delta p}{\rho \cdot g \cdot L} = \frac{Z_t}{L} \Rightarrow \Delta p = Z_t \cdot \rho \cdot g$

$$Z_t = \frac{\lambda \cdot L \cdot v^2}{D \cdot 2g}$$

$$i_e = \frac{Z_t}{L} = \frac{\lambda}{D} \cdot \frac{1}{2} \cdot \rho \cdot v^2$$
friction slope \[ i_E = a v^b \]

- LF: laminar flow
- TF: transitional zone
- Quadratic zone
- \( v_k \leftrightarrow Re_k = 2320 \)
Effect of dimensions of projections on flow in close vicinity to pipeline wall
**FRICION FACTOR $\lambda$**

1. **Laminar flow**
   - Poiseuille $\lambda = \frac{64}{Re}$

2. **Region of transition**

3. **Smooth pipes**
   - Blasius $\lambda = \frac{0.3164}{Re^{0.25}}$

4. **Turbulent flow - transitional zone**
   - Colebrook - White
     $$\frac{1}{\sqrt{\lambda}} = -2\log\left(\frac{2.51}{Re\sqrt{\lambda}} + \frac{\Delta}{3.71D}\right)$$

5. **Rough turbulent quadratic zone**
   - Altshul $\lambda = 0.11\left(\frac{68}{Re} + \frac{\Delta}{D}\right)^{0.25}$
   - Nikuradse $\lambda = \frac{0.25\left(\log\frac{3.71D}{\Delta}\right)^2}{\sqrt{\lambda} \Delta}$

$Re > \frac{200D}{\sqrt{\lambda} \Delta}$
Notes:

- tap water $T = 12^\circ C$ $\rightarrow \nu = 1,24 \cdot 10^{-6} \text{ m}^2\text{s}^{-1}$

- hydraulic roughness $\Delta = 0,01 \text{ mm} \div 1 \text{ mm}$

- recommended velocity for water pipeline $0,5 \div 1,5 \text{ ms}^{-1}$
LOCAL LOSSES IN PIPELINES

Total losses = friction losses + local losses

\[ Z = Z_t + Z_m = \sum \left( \lambda_j \frac{L_j}{D_j} + \sum \xi_i \right) \frac{v_j^2}{2g} \]

INLET (common, curved, streamlined, suction basket, backflow valve, grid, …)

\[ \xi_v = 0.5 \quad 0.05 \div 0.2 \quad 2 \div 20 \]
CHANGE OF DIRECTION (bend, knee pipe, segment knee pipe)

\[ \zeta_s = f\left(\frac{r_s}{D}, \alpha, \frac{\Delta}{D}\right) \]

BRANCHING, CONNECTION

\[ \zeta_o = f\left(\frac{D_1}{D_3}, \frac{D_2}{D_3}, \frac{Q_2}{Q_1}, \frac{Q_3}{Q_1}, \alpha\right) \]

CHANGE OF PROFILE (sudden x continuous, enlargement x thinning)

\[ \zeta_z = f\left(\frac{D_1}{D_2}, \delta\right) \]

MEASURING DEVICE (pipe flow meters)

OUTLET

\[ \zeta_u = 0 \]

large reservoir \[ \zeta_u = 1 \]
CLOSURES (valves, backflow valves, taps, slide valves)

\[ \zeta_u = f(\text{type, opening}) \]
HYDRAULIC CALCULATIONS OF PIPELINES

3 kinds of equations:

- Bernoulli equation ← elevations and pressure relations, boundary conditions
- continuity equation
- equation of losses

geometry and roughness of pipe, discharge

calculation: Q, v, D, L, H, p, Z
BE
\[ H + \frac{p_A}{\rho g} + \alpha \frac{v_A^2}{2g} = \frac{p_B}{\rho g} + \alpha \frac{v_B^2}{2g} + \sum Z \]

CE
\[ Q = v_1 \cdot S_1 = v_2 \cdot S_2 = v_j \cdot S_j \]

equation of losses
\[ \sum Z = \sum Z_t + \sum Z_m = \sum \left( \lambda \frac{L}{D} + \sum \zeta_i \right) \frac{v^2}{2g} \]
Note:
so called „hydraulically long pipeline“

\[ Z_m \ll Z_t \implies Z = Z_t \]

\[ \frac{\alpha v^2}{2g} \ll Z_t \implies \frac{\alpha v^2}{2g} \approx 0 \implies EL \equiv PL \]
• **open and large reservoirs**

\[ p_A = p_B = 0 \] (overpressures)
\[ v_A = v_B = 0 \]

BE: \[ H = \sum Z \]
... total head loss
exit loss

\[ \sum_+ = \sum Z \]

\[ H = \sum Z \]

• **outlet from pipe to open space**

\[ p_B = 0 \]

BR: \[ H = \frac{\alpha v^2}{2g} + \sum Z \]

no exit loss
UNDERPRESSURES IN PIPELINES

\[ \left| p_{va} \right|_{\text{max}} \approx (6 \div 8).10^4 \text{ Pa} \]

underpressures \( \rightarrow \) cavitation

\[ \left| \frac{p_{va}}{\rho g} \right|_{\text{max}} \approx (6 \div 8) \text{ m w. col.} \]

always to check!

- places with high elevation
  siphon tube
  (top above upper reservoir level)

BE 0-1:

\[ H + \frac{p_0}{\rho g} + \frac{\alpha v_0^2}{2g} = s + \frac{p_1}{\rho g} + \frac{\alpha v_1^2}{2g} + \sum Z \]

\( H = 0, \ v_0 = 0, \ p_0 = p_a \ \Rightarrow \)

\[ \Rightarrow \frac{p_1}{\rho g} = \frac{p_a}{\rho g} - s - \frac{\alpha v_1^2}{2g} - \sum Z \]

for \[ \frac{p_{va}}{\rho g} \]

\( \Rightarrow s_{\text{max}} \)
**Description of system:**
lower reservoir → suction pipe **SP** → pump **Č** → delivery pipe **VP** → upper reservoir (event. outlet to open space)

**Types of losses:**

**SP:**
$Z_s \rightarrow$ friction, suction basket, backflow valve, knee pipe, bends

**VP:**
$Z_v \rightarrow$ friction, closures, knee pipe, bends

**Solved problems:**
- determination of $Q$, $D$
- checking of underpressure in **SP**
- determination of input power of **Č**
determination of $Q$, $D$ - basic equations $(BE, CE)$

for open large reservoir

$$p_A = p_a \quad (p_{pA} = 0), \quad v_A \approx 0$$

checking of underpressure in $SP$

$BE$: lower reservoir - Č

$$\frac{p_a}{\rho g} = H_{gs} + \frac{p_{\tilde{c}}}{\rho g} + \frac{\alpha v_s^2}{2g} + Z_s$$

$$H_{va} = \left| \frac{p_{va}}{\rho g} \right| = \left| \frac{p_a - p_{\tilde{c}}}{\rho g} \right| = H_{gs} + Z_S + \frac{\alpha v_s^2}{2g}$$

practically $H_{va} < (6 \div 8)$ m w. col.
geodetic gradient: \( H_g = H_{gs} + H_{gv} \)

total head:
\[
H_d = H_g + Z_S + Z_V + \frac{p_B - p_A}{\rho g} + \frac{v_B^2 - v_A^2}{2g}
\]

For open large reservoirs

\( p_A = p_B = p_a, \quad v_A = v_B \approx 0 \)

\[
H_d = H_g + Z_S + Z_V = H_g + Z
\]

Input power
\[
P = \frac{1}{\eta} \rho g Q H_d \quad [W]
\]

efficacy: \( \eta \) … practically up to 0.9
PIPELINE SYSTEMS

- branched
- close loop
- combined

Branched net

Closed loop

V - distribution reservoir
WATER HAMMER

quick manipulation with closure, pump starting or failure

\[\downarrow\]

rapid change of pressure (water hammer),
in particular at long pipelines

protection against water hammer:
• sufficiently slow manipulation with closures
• air chambers, surge chambers (equilibrating towers)
• ...