Hydraulics of pipelines
Application of Bernoulli equation **BE**
continuity equation **CE**

**BE 1 - 2**
\[ h_1 + \frac{p_1}{\rho g} + \frac{\alpha v_1^2}{2g} = h_2 + \frac{p_2}{\rho g} + \frac{\alpha v_2^2}{2g} + Z \]

\( Z \) – loss head (losses):
- **friction losses** \( Z_t \) (in distance \( L \))
- **local losses** \( Z_m \)
\[ Z = Z_t + Z_m \]

**CE 1 - 2**
\[ Q_1 = Q_2 \Rightarrow v_1 \cdot S_1 = v_2 \cdot S_2 \]
FRICTION LOSSES

uniform flow $\varnothing D = \text{const}, \, v = \text{const}$

$i_E = \text{hydraulic slope (friction slope)} = \text{slope of EL}$

$Z_t = Z_t (v, D, L, \text{character of walls of pipeline})$
Equilibrium of forces in elementary volume EV

\[(p + dp) \cdot S - p \cdot S = \tau_0 \cdot O \cdot ds\]

\[\frac{dp}{ds} = \tau_0 \cdot \frac{O}{S}\]

for circle pipeline \(\varnothing D\):

\[O = \pi \cdot D, \quad S = \frac{\pi \cdot D^2}{4} \Rightarrow \frac{O}{S} = \frac{\pi \cdot D}{\pi \cdot D^2} = \frac{4}{D}\]

\[\frac{dp}{ds} = \tau_0 \cdot \frac{4}{D}\]
Expression of $\tau_0$ under turbulent flow

friction coefficient:

$$f_f = \frac{\tau_0}{\text{kinetic energy in unit volume}}$$

$$\lambda = 4 \cdot f_f = \frac{8 \cdot \tau_0}{\rho \cdot v^2} \Rightarrow \tau_0 = \frac{\lambda}{8} \cdot \rho \cdot v^2$$

coefficient of friction loss

$$\lambda = f (\text{Re}, \Delta/D)$$

Darcy-Weisbach equation

$$i = \frac{\text{dp}}{\text{ds}} = \frac{\tau_0 \cdot 4}{D} = \frac{\lambda}{8} \cdot \rho \cdot v^2 \cdot \frac{4}{D} = \frac{\lambda}{2} \cdot \rho \cdot v^2$$

uniform flow:

(for $\text{ds} = L$, $\text{dp} = \Delta p$)

$$\text{PL} \parallel \text{EL} \Rightarrow \frac{\Delta p}{\rho \cdot g \cdot L} = \frac{Z_t}{L} \Rightarrow \Delta p = Z_t \cdot \rho \cdot g$$

$$Z_t = \frac{\lambda \cdot L \cdot v^2}{D \cdot 2g}$$

$$i_E = \frac{Z_t}{L} = \frac{\lambda}{D \cdot 2g} \cdot \frac{1}{2} \cdot v^2$$
friction slope \( i_E = a v^b \)

- \( b = 1 \) LF (transit zone)
- \( 1 < b < 2 \) TF
- \( b = 2 \) quadratic zone

\[ v_k \leftrightarrow Re_k = 2320 \]

LP - laminar flow
TP - turbulent flow
Effect of dimensions of projections on flow in close vicinity to pipeline wall

![Diagram showing laminar and turbulent flow with labels u_max and δ for each flow type.]

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Hydraulics of pipelines
**FRICTION FACTOR \( \lambda \)**

1. **Laminar flow**
   
   Poiseuille \( \lambda = \frac{64}{Re} \)

2. **Region of transition**

3. **Smooth pipes**
   
   Blasius \( \lambda = \frac{0.3164}{Re^{0.25}} \)

4. **Turbulent flow - transitional zone**
   
   Colebrooke - White
   
   \[
   \frac{1}{\sqrt{\lambda}} = -2\log\left( \frac{2.51}{Re\sqrt{\lambda}} + \frac{\Delta}{3.71D} \right)
   \]

5. **Rough turbulent quadratic zone**

   Altshul \( \lambda = 0.11 \left( \frac{68}{Re} + \frac{\Delta}{D} \right)^{0.25} \)

   \[
   Re > \frac{200 \ D}{\sqrt{\lambda} \ \Delta}
   \]

   Nikuradse \( \lambda = \frac{0.25}{\left( \log \frac{3.71D}{\Delta} \right)^2} \)
Notes:

- tap water $T = 12^\circ C \rightarrow v = 1,24 \times 10^{-6} \text{ m}^2\text{s}^{-1}$

- hydraulic roughness $\Delta = 0,01 \text{ mm} \div 1 \text{ mm}$

- recommended velocity for water pipeline $0,5 \div 1,5 \text{ ms}^{-1}$
LOCAL LOSSES IN PIPELINES

Z_{mi} = \xi_i \cdot \frac{v^2}{2g} \quad \text{loss coefficient } \xi_i

Total losses = friction losses + local losses

Z = Z_t + Z_m = \sum \left( \lambda_j \frac{L_j}{D_j} + \sum \xi_i \right) \frac{v_j^2}{2g}

INLET (common, curved, streamlined, suction basket, backflow valve, grid, …)

\xi_v = 0,5 \quad 0,05 \div 0,2 \quad 2 \div 20

Hydraulics of pipelines
CHANGE OF DIRECTION (bend, knee pipe, segment knee pipe)

\[ \zeta_s = f \left( \frac{r_s}{D}, \alpha, \frac{A}{D} \right) \]

BRANCHING, CONNECTION

\[ \zeta_o = f \left( \frac{D_1}{D_3}, \frac{D_2}{D_3}, \frac{Q_2}{Q_1}, \frac{Q_3}{Q_1}, \alpha \right) \]

CHANGE OF PROFILE (sudden x continuous, enlargement x thinning)

\[ \zeta_z = f \left( \frac{D_1}{D_2}, \delta \right) \]

MEASURING DEVICE (pipe flow meters)

OUTLET

\[ \zeta_u = 0 \]

large reservoir

\[ \zeta_u = 1 \]
CLOSURES (valves, backflow valves, taps, slide valves)

\[ \zeta_u = f(\text{type, opening}) \]
HYDRAULIC CALCULATIONS OF PIPELINES

3 kinds of equations:

- Bernoulli equation $\leftrightarrow$ elevations and pressure relations, boundary conditions
- continuity equation
- equation of losses $\left\{ \right\} \leftarrow$ geometry and roughness of pipe, discharge

calculation: $Q, v, D, L, H, p, Z$
BE

\[ H + \frac{p_A}{\rho g} + \frac{\alpha v_A^2}{2g} = \frac{p_B}{\rho g} + \frac{\alpha v_B^2}{2g} + \sum Z \]

CE

\[ Q = v_1 \cdot S_1 = v_2 \cdot S_2 = v_j \cdot S_j \]

equation of losses

\[ \sum Z = \sum Z_t + \sum Z_m = \sum \left( \lambda \frac{L}{D} + \sum \zeta_i \right) \frac{v^2}{2g} \]
Note:
so called „hydraulically long pipeline“.

\[ Z_m \ll Z_t \implies Z = Z_t \]
\[ \frac{\alpha v^2}{2g} \ll Z_t \implies \frac{\alpha v^2}{2g} \approx 0 \implies EL \equiv PL \]
- **open and large reservoirs**

  \[ H = \sum Z \]

  ... total head loss

  \[ p_A = p_B = 0 \] (overpressures)

  \[ v_A = v_B = 0 \]

- **outlet from pipe to open space**

  \[ p_B = 0 \]

  \[ H = \frac{\alpha v_k^2}{2g} + \sum Z \]

  no exit loss

\[ \phi D_1, L_1, v_1, Q \]

\[ \phi D_2, L_2, v_2, Q \]
UNDERPRESSURES IN PIPELINES

\[ |p_{va}|_{\text{max}} \approx (6 \div 8) \times 10^4 \text{ Pa} \]

underpressures → cavitation

\[ \frac{|p_{va}|}{\rho g}_{\text{max}} \approx (6 \div 8) \text{ m w. col.} \]

always to check!

- places with high elevation

  siphon tube

  (top above upper reservoir level)

\[ BE \ 0-1: \]

\[ h_0 + \frac{p_0}{\rho g} + \frac{\alpha v_0^2}{2g} = h_1 + \frac{p_1}{\rho g} + \frac{\alpha v_1^2}{2g} + Z_{0-1} \]

\[ h_0 = 0, \ v_0 = 0, \ p_0 = p_a, \ h_1 = s \implies \]

\[ \implies \frac{p_1}{\rho g} = \frac{p_a}{\rho g} - s - \frac{\alpha v_1^2}{2g} - Z_{0-1} \]

for \[ \left| \frac{p_{va}}{\rho g} \right|_{\text{max}} \implies s_{\text{max}} \]
Description of system:
lower reservoir → suction pipe SP → pump Č → delivery pipe VP → upper reservoir (event. outlet to open space)

Types of losses:
SP:
$Z_s$ → friction, suction basket, backflow valve, knee pipe, bends

VP:
$Z_v$ → friction, closures, knee pipe, bends

Solved problems:
- determination of $Q$, $D$
- checking of underpressure in SP
- determination of input power of Č
determination of $Q$, $D$ - basic equations (BE, CE)

for open large reservoir

$p_A = p_a \quad (p_{PA} = 0), \quad v_A \approx 0$

checking of underpressure in SP

BE: lower reservoir - $\check{C}$

\[
\frac{p_a}{\rho g} = H_{gs} + \frac{p_{\check{C}}}{\rho g} + \frac{\alpha v_s^2}{2g} + Z_s
\]

\[
H_{va} = \frac{p_{va}}{\rho g} = \frac{p_a - p_{\check{C}}}{\rho g} = H_{gs} + Z_s + \frac{\alpha v_s^2}{2g}
\]

practically $H_{va} < (6 \div 8)$ m w. col.
determination of necessary input power

geodetic gradient: \( H_g = H_{gs} + H_{gv} \)
total head:
\[
H_d = H_g + Z_S + Z_v + \frac{p_B - p_A}{\rho g} + \frac{v_B^2}{2g} - \frac{v_A^2}{2g}
\]
For open large reservoirs
\( p_A = p_B = p_a, \ v_A = v_B \approx 0 \)
\[
H_d = H_g + Z_S + Z_v = H_g + Z
\]
Input power
\[
P = \frac{1}{\eta} \rho g Q H_d \quad [W]
\]
efficacy: \( \eta \) … practically up to 0,9
PIPELINE SYSTEMS

- branched
- close loop
- combined

Branched net

V - distribution reservoir

Closed loop
WATER HAMMER

quick manipulation with closure, pump starting or failure

\[ \downarrow \]

rapid change of pressure (water hammer),
in particular at long pipelines

protection against water hammer:

• sufficiently slow manipulation with closures
• air chambers, surge chambers (equilibrating towers)
• ...