Solved problems – 8th exercise

Solved problem 8.1

A rectangular concrete drive channel was constructed to conduct water to small hydro-electric power plant. Concrete of both bed and walls of the channel has been done in a current way. Width of the channel bed is $\boldsymbol{b}$ = 2,0 m, longitudinal slope of channel bed $\boldsymbol{i_0}$ = 0,6 ‰. What discharge will flow to the hydro-power plant, supposing that flow in the channel is uniform? Depth of water in channel should be $\boldsymbol{y_0}$ = 1,20 m.

[Results: 2,8 $m^3.s^{-1}$]

S o l u t I o n

Rectangular area $S = b.y$ = 2,0 . 1,2 = 2,4 m^2.
Wetted perimeter $O = b + 2.y$ = 2,0 + 2 . 1,2 = 4,4 m.
Hydraulic radius $R = S / O$ = 0,545 m.
Slope 0,6 ‰ – for calculation it has to be expressed as a dimensionless number, i.e. i_0 = 0,0006.

Velocity can be calculated from Chezy equation $v = C \cdot \sqrt{Ri}$ and consequently, discharge will be calculated from continuity equation $Q = C \cdot S \cdot \sqrt{Ri}$

Velocity coefficient *C* can be calculated e.g. from Manning formula. Value of roughness coefficient *n* will be determined from tab. 12 – usual value for concrete channels should be *n* = 0,014.

velocity coefficient $$C = \frac{1}{n} \cdot R^{1/6} = \frac{1}{0{,}014}\ 0{,}545^{\frac{1}{6}} = 64{,}5\ m^{0,5}.s^{-1}$$

velocity from Chezy equation $$v = C\sqrt{R\,.\,i_e}$$

discharge from continuity equation $$Q = S \cdot v$$

$$Q = C \cdot S \cdot \sqrt{Ri} \quad = \quad 64{,}5 \cdot 2{,}4 \cdot \sqrt{0{,}545 \cdot 0{,}0006} = 2{,}80\ m^3.s^{-1}.$$

Solved problem 8.2

Calculate the depth y_o in which the discharge Q = 1,5 m^3s^{-1} will flow through the channel with trapezoid-shaped cross section. Width of the channel bed is b = 2 m, slope of bed i_o = 0,05 %, slope of sides is 1 : 1,5. The channel is excavated in gravel sends, representative grain size was determined from grain size curve d_e = 0,02 m, Determine the roughness coefficient from Strickler formula.

[Results: 0,8 m]

Solution:

Strickler formula: $\frac{1}{n} = \frac{21,1}{k_s^{1/6}}$, where $k_s \sim d_e$. Solving the formula $\rightarrow$ n = 0, 0247.

Step-by-step method will be used to determine depth $y_o \rightarrow$ discharge **Q** is calculated for elective depths till there is an accordance of given and calculated discharge

y_o	**S**	**O**	**R**	**C**	**v**	**Q**	
1,0	3,500	5,610	0,624	37,44	0,661	2,314	"mistake"
0,5	1,375	3,805	0,361	34,17	0,459	0,631	"mistake"
0,8	2,560	4,888	0,524	36,36	0,588	1,506	"accord."

Discharge Q = 1,5 m^3s^{-1} flows through the channel with the depth $y_o \cong$ 0,8 m.

Equations to calculate the table:

area of trapezoid-shaped profile: $S = y_0.(b+m.y_0)$, where m expresses bank slope 1:m, i.e. here m = 1,5,

wetted perimeter of trapezoid-shaped profile: $O = b + 2.y_o\sqrt{1+m^2}$

hydraulic radius: $R = S/O$,

velocity coefficient: $C = \frac{1}{n} \cdot R^{1/6}$

mean velocity: $v = C\sqrt{R.i_e}$

discharge: $Q = v.S$

Note:

The same procedure of calculation will be used to design a channel width if - under given roughness, longitudinal channel slope and bank slope - discharge Q is to flow in the channel with required channel depth y.

Solved problem 8.3

Determine diameter ***D*** of a circular conduit in such a way that discharge ***Q*** = 6,5 m^3s^{-1} will flow through it with a free water level. Values of diameters of produced profiles vary after 200 mm. Longitudinal slope of conduit i_0 = 0,003, roughness coefficient ***n*** = 0,011. Determine the maximum depth y_O and velocity of flow ***v***.

What longitudinal slope i_0 [‰] should have the conduit in order to the indicated discharge was the maximum one in conduit.

S o l u t i o n:

Discharge *Q* is to flow through the conduit with free water level. In order to the conduit is suggested in the most economic way, it is necessary to choose from the production range such a diameter – minimum diameter – which will have the capacity to conduct requested discharge with free level.

Diameter *D* can be calculated from Chezy equation + continuity equation ($Q = C \cdot S \cdot \sqrt{Ri}$) will be used, supposing that the requested discharge is considered to be a capacity one (related to full profile) Q_D.

To solve Chezy and continuity equations, velocity coefficient **C** can be calculated from Manning formula. Hydraulic radius $R = \frac{D}{4}$

$$6{,}5 = C S \sqrt{R \cdot i_o} = \frac{1}{n} \cdot \frac{\pi D^2}{4} \cdot \left(\frac{D}{4}\right)^{2/3} \cdot i_o^{1/2} = \frac{1}{n} \cdot \pi \cdot \frac{D^{8/3}}{4^{5/3}} \cdot i_o^{1/2}$$

Solving this equation, the result will be *D* = 1,711m. The proximate larger diameter from the production range will be selected ⇒ suggested diameter is D_n = 1,8 m and its cross sectional area *S* = 2,5434 m^2.

Review of suggested profile:

Under uniform flow and full circular profile

$$v_{D,n} = C_{D,n} \cdot \sqrt{R_{D,n} \cdot i_o} = 2{,}92 \ ms^{-1}$$

$$Q_{D,n} = C_{D,n} . S_{D,n} \sqrt{R_{D,n} \cdot i_o} = 7{,}441 \ m^3s^{-1}$$

For discharge *Q* = 6,5 m^3s^{-1} which will flow through the conduit, the ratio

$Q / Q_{D,n}$ = 6,5 / 7,441 = 0,874.

For this relative value of discharge, also relative values of depth and velocity can be determined from tab. 13 .

y / D_n = 0,7156 v / v_{Dn} = 1,1417

From these values

y_O = 0,7156·1,8 = 1,288 m , v = 1,1417·2,92 = 3,33 $m \cdot s^{-1}$.

Close profile, when full, does not carry the maximum discharge, but the maximum discharge is reached at a smaller depth (see the graph of tab. 13). To determine longitudinal slope i_O at which the discharge Q = 6,5 m^3s^{-1} will be maximum one, Chezy equation and continuity equations will be used again:

$$i_o = \frac{Q^2}{C^2.S^2.R} = \frac{Q^2}{K^2};\quad \text{(C from Manning formula)}$$

Conveyance at full profile (i.e. when y = D):

$$K_D = C_D \cdot S_D \cdot \sqrt{R_D} = \frac{1}{n} \cdot \left(\frac{D}{4}\right)^{1/6} \cdot \frac{\pi D^2}{4} \cdot \left(\frac{D}{4}\right)^{1/2} = \frac{1}{n} \cdot \frac{\pi}{8.4^{1/6}} D^{8/3} = 135{,}85\ m^3s^{-1}$$

From tab. 13: for Q_{max} and K_{max} (i.e. at relative depth $\frac{y}{D} = 0{,}95$) $\rightarrow$

$\rightarrow \frac{K}{K_D} = 1{,}087$ $\quad\Rightarrow\quad$ K = 1,087·K_D = 1,087·135,85 = 147,67 m^3s^{-1}.

From Chezy equation $Q = C \cdot S \cdot \sqrt{R \cdot i} = K \cdot \sqrt{i} \Rightarrow i = \frac{Q^2}{K^2}$ = 0,00194 = 1,94 ‰.

Solved problem 8.4

Discharge $\boldsymbol{Q}$ = 12 m^3s^{-1} flows through rectangular channel. Width of the channel is $\boldsymbol{b}$ = 3,0 m. Calculate and draw in graph a dependency of energy head (specific energy) of cross section on channel depth $\boldsymbol{E_d = f(y)}$. Find out the value of critical depth. Determine kind of flow in the channel for two depths: 0,6 m and 2,4 m.

[Results: 1,18 m; super-critical flow, sub-critical flow]

S o l u t I o n:

Velocity in channel – from continuity equation: $v = \frac{Q}{S} = \frac{12}{b.y} = \frac{4}{y}$

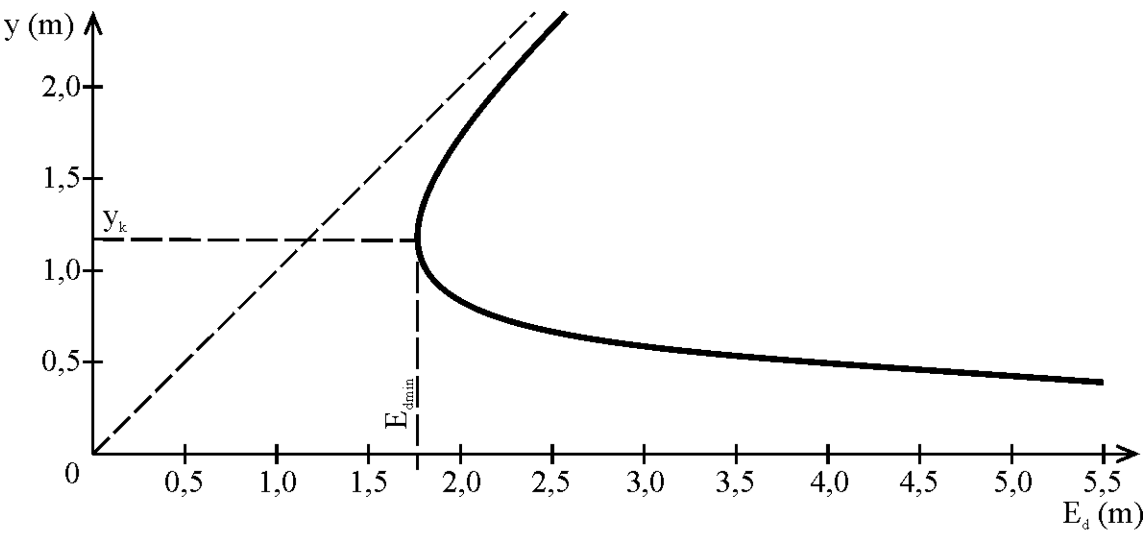
Specific energy of cross section: $E_d = y + \frac{\alpha . v^2}{2g}$

$$\Rightarrow \quad E_d = y + \frac{(4/y)^2}{2g} = y + \frac{0{,}815}{y^2} \qquad \text{(estimated } \alpha = 1{,}0\text{)}$$

E_d is calculated for elective depths:

y (m)	0,4	0,6	0,8	1,0	1,2	1,4	1,6	1,8	2,0	2,2	2,4
E_d (m)	5,50	2,87	2,07	1,82	1,77	1,82	1,92	2,05	2,20	2,37	2,54

Course of the dependency can be seen at graph:



Result coming from the graphical dependency: $y_k \cong 1{,}20$ m.
More precise solution: for rectangular cross section, using the formula derived from general condition of critical flow

$$y_k = \sqrt[3]{\frac{\alpha \cdot Q^2}{g \cdot b^2}} = 1{,}18 \ m$$

As y = 0,6 m $< y_k$ = 1,18 m → flow in the channel is supercritical.
Under the depth y = 2,4 m $> y_k$ → flow in the channel is subcritical.