Solved problems – 7\textsuperscript{th} exercise

Solved problem 7.1

A rectangular concrete drive channel was constructed to conduct water to small hydro-electric power plant. Concrete of both bed and walls of the channel has been done in a current way. Width of the channel bed is $b = 2.0$ m, longitudinal slope of channel bed $i_0 = 0.6 \text{‰}$. What discharge will flow to the hydro-power plant, supposing that flow in the channel is uniform? Depth of water in channel should be $y_0 = 1.20$ m.

[Results: $2.8 \text{ m}^3 \text{.s}^{-1}$]

Solution

Rectangular area $S = b \cdot y = 2.0 \cdot 1.2 = 2.4$ m\textsuperscript{2}.
Wetted perimeter $O = b + 2 \cdot y = 2.0 + 2 \cdot 1.2 = 4.4$ m.
Hydraulic radius $R = S / O = 0.545$ m.
Slope 0.6 ‰ – for calculation it has to be expressed as a dimensionless number, i.e. $i_0 = 0.0006$.

Velocity can be calculated from Chezy equation $v = C \cdot \sqrt{Ri}$ and consequently, discharge will be calculated from continuity equation $Q = C \cdot S \cdot \sqrt{Ri}$.

Velocity coefficient $C$ can be calculated e.g. from Manning formula. Value of roughness coefficient $n$ will be determined from tab. 12 – usual value for concrete channels should be $n = 0.014$.

\[
\text{velocity coefficient } C = \frac{1}{n} \cdot R^{1/6} = \frac{1}{0.014} \cdot 0.545^{5/6} = 64.5 \text{ m}^{0.5} \text{.s}^{-1}
\]

\[
\text{velocity from Chezy equation } v = C \sqrt{R \cdot i_0}
\]

\[
\text{discharge from continuity equation } Q = S \cdot v
\]

\[
Q = C \cdot S \cdot \sqrt{Ri} = 64.5 \cdot 2.4 \cdot \sqrt{0.545 \cdot 0.0006} = 2.80 \text{ m}^3 \text{.s}^{-1}.
\]
Solved problem 7.2

Calculate the depth $y_0$ in which the discharge $Q = 1,5 \, \text{m}^3\text{s}^{-1}$ will flow through the channel with trapezoid-shaped cross section. Width of the channel bed is $b = 2 \, \text{m}$, slope of bed $i_o = 0,05 \%$, slope of sides is $1 : 1,5$. The channel is excavated in gravel sends, representative grain size was determined from grain size curve $d_e = 0,02 \, \text{m}$. Determine the roughness coefficient from Strickler formula.

[Results: 0,8 m]

Solution:

Strickler formula: $\frac{1}{n} = \frac{21,1}{k_s^{1/6}}$, where $k_s \sim d_e$. Solving the formula $\rightarrow n = 0,0247$.

Step-by-step method will be used to determine depth $y_0 \rightarrow$ discharge $Q$ is calculated for elective depths till there is an accordance of given and calculated discharge

<table>
<thead>
<tr>
<th>$y_0$</th>
<th>S</th>
<th>O</th>
<th>R</th>
<th>C</th>
<th>v</th>
<th>Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,0</td>
<td>3,500</td>
<td>5,610</td>
<td>0,624</td>
<td>37,44</td>
<td>0,661</td>
<td>2,314</td>
</tr>
<tr>
<td>0,5</td>
<td>1,375</td>
<td>3,805</td>
<td>0,361</td>
<td>34,17</td>
<td>0,459</td>
<td>0,631</td>
</tr>
<tr>
<td>0,8</td>
<td>2,560</td>
<td>4,888</td>
<td>0,524</td>
<td>36,36</td>
<td>0,588</td>
<td>1,506</td>
</tr>
</tbody>
</table>

Discharge $Q = 1,5 \, \text{m}^3\text{s}^{-1}$ flows through the channel with the depth $y_0 = 0,8 \, \text{m}$.

Equations to calculate the table:

area of trapezoid-shaped profile: $S = y_0 \cdot (b+m \cdot y_0)$, where $m$ expresses bank slope 1:m, i.e. here $m = 1,5$,

wetted perimeter of trapezoid-shaped profile: $O = b + 2 \cdot y_0 \cdot \sqrt{1 + m^2}$

hydraulic radius: $R = \frac{S}{O}$,

velocity coefficient: $C = \frac{1}{n} \cdot R^{1/6}$

mean velocity: $v = C \cdot \sqrt{R \cdot i_e}$

discharge: $Q = v \cdot S$

Note:

The same procedure of calculation will be used to design a channel width if - under given roughness, longitudinal channel slope and bank slope - discharge $Q$ is to flow in the channel with required channel depth $y$. 

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Solved problem 7.3

Determine diameter $D$ of a circular conduit in such a way that discharge $Q = 6,5 \text{ m}^3\text{s}^{-1}$ will flow through it with a free water level. Values of diameters of produced profiles vary after 200 mm. Longitudinal slope of conduit $i_0 = 0,003$, roughness coefficient $n = 0,011$. Determine the maximum depth $y_o$ and velocity of flow $v$.

What longitudinal slope $i_0 [\%]$ should have the conduit in order to the indicated discharge was the maximum one in conduit.

Solution:

Discharge $Q$ is to flow through the conduit with free water level. In order to the conduit is suggested in the most economic way, it is necessary to choose from the production range such a diameter – minimum diameter – which will have the capacity to conduct requested discharge with free level.

Diameter $D$ can be calculated from Chezy equation + continuity equation ($Q = C \cdot S \cdot \sqrt{R \cdot i_o}$) will be used, supposing that the requested discharge is considered to be a capacity one (related to full profile) $Q_D$.

To solve Chezy and continuity equations, velocity coefficient $C$ can be calculated from Manning formula. Hydraulic radius $R = \frac{D}{4}$

$$6,5 = C \cdot S \cdot \sqrt{R \cdot i_o} = \frac{1}{n} \cdot \frac{\pi D^2}{4} \cdot \left(\frac{D}{4}\right)^{2/3} \cdot i_o^{1/2} = \frac{1}{n} \cdot \frac{\pi D^{8/3}}{4^{5/3}} \cdot i_o^{1/2}$$

Solving this equation, the result will be $D = 1,711 \text{ m}$. The proximate larger diameter from the production range will be selected $\Rightarrow$ suggested diameter is $D_n = 1,8 \text{ m}$ and its cross sectional area $S = 2,5434 \text{ m}^2$.

Review of suggested profile:

Under uniform flow and full circular profile

$$v_{D,n} = C_{D,n} \sqrt{R_{D,n} \cdot i_o} = 2,92 \text{ m} \text{s}^{-1}$$

$$Q_{D,n} = C_{D,n} \cdot S_{D,n} \sqrt{R_{D,n} \cdot i_o} = 7,441 \text{ m}^3\text{s}^{-1}$$

For discharge $Q = 6,5 \text{ m}^3\text{s}^{-1}$ which will flow through the conduit, the ratio $Q / Q_{D,n} = 6,5 / 7,441 = 0,874$.

For this relative value of discharge, also relative values of depth and velocity can be determined from tab. 13 .

$$y / D_n = 0,7156 \quad v / v_{D_n} = 1,1417$$

From these values

$$y_o = 0,7156 \cdot 1,8 = 1,288 \text{ m}, \quad v = 1,1417 \cdot 2,92 = 3,33 \text{ m} \text{s}^{-1}.$$
Close profile, when full, does not carry the maximum discharge, but the maximum discharge is reached at a smaller depth (see the graph of tab. 13). To determine longitudinal slope \( i_0 \) at which the discharge \( Q = 6.5 \text{ m}^3\text{s}^{-1} \) will be maximum one, Chezy equation and continuity equations will be used again:

\[
i_0 = \frac{Q^2}{C^2 \cdot S^2 \cdot R} = \frac{Q^2}{K^2} \quad \text{(C from Manning formula)}
\]

Conveyance at full profile (i.e. when \( y = D \)):

\[
K_D = C_D \cdot S_D \cdot \sqrt[4]{R_D} = \frac{1}{n} \left( \frac{D}{4} \right)^{1/6} \cdot \frac{\pi D^2}{4} \cdot \left( \frac{D}{4} \right)^{1/2} = \frac{1}{n} \cdot \frac{\pi}{8.4^{1/6}} D^{8/3} = 135.85 \text{ m}^3\text{s}^{-1}
\]

From tab. 13: for \( Q_{\text{max}} \) and \( K_{\text{max}} \) (i.e. at relative depth \( \frac{y}{D} = 0.95 \) →

\[
\rightarrow \frac{K}{K_D} = 1.087 \quad \Rightarrow \quad K = 1.087 \cdot K_D = 1.087 \cdot 135.85 = 147.67 \text{ m}^3\text{s}^{-1}.
\]

From Chezy equation \( Q = C \cdot S \cdot \sqrt[4]{R} \cdot i = K \cdot \sqrt[i]{i} \Rightarrow i = \frac{Q^2}{K^2} = 0.00194 = 1.94 \text{‰} \).
**Solved problem 7.4**

Discharge \( Q = 12 \text{ m}^3\text{s}^{-1} \) flows through rectangular channel. Width of the channel is \( b = 3.0 \text{ m} \). Calculate and draw in graph a dependency of energy head (specific energy) of cross section on channel depth \( E_d = f(y) \). Find out the value of critical depth. Determine kind of flow in the channel for two depths: 0.6 m and 2.4 m.

[Results: 1.18 m; super-critical flow, sub-critical flow]

**Solution:**

Velocity in channel – from continuity equation:

\[
v = \frac{Q}{S} = \frac{12}{b \cdot y} = \frac{4}{y}
\]

Specific energy of cross section:

\[
E_d = y + \frac{\alpha \cdot v^2}{2g}
\]

\[
\Rightarrow E_d = y + \frac{(4/y)^2}{2g} = y + \frac{0.815}{y^2} \quad \text{(estimated } \alpha = 1.0)\]

\( E_d \) is calculated for elective depths:

<table>
<thead>
<tr>
<th>( y ) (m)</th>
<th>0.4</th>
<th>0.6</th>
<th>0.8</th>
<th>1.0</th>
<th>1.2</th>
<th>1.4</th>
<th>1.6</th>
<th>1.8</th>
<th>2.0</th>
<th>2.2</th>
<th>2.4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E_d ) (m)</td>
<td>5.50</td>
<td>2.87</td>
<td>2.07</td>
<td>1.82</td>
<td>1.77</td>
<td>1.82</td>
<td>1.92</td>
<td>2.05</td>
<td>2.20</td>
<td>2.37</td>
<td>2.54</td>
</tr>
</tbody>
</table>

Course of the dependency can be seen at graph:

![Graph of specific energy head](image)

Result coming from the graphical dependency: \( y_k \approx 1.20 \text{ m} \).

More precise solution: for rectangular cross section, using the formula derived from general condition of critical flow

\[
y_k = \sqrt[3]{\frac{\alpha \cdot Q^2}{g \cdot b^2}} = 1.18 \text{ m}
\]

As \( y = 0.6 \text{ m} < y_k = 1.18 \text{ m} \) → flow in the channel is supercritical.

Under the depth \( y = 2.4 \text{ m} > y_k \) → flow in the channel is subcritical.