Solved problems – 7th exercise

Solved problem 7.1

In the system of tanks at fig. 1 there are cross walls with outlets. The first outlet is square-shaped with the area $\boldsymbol{S_1}$ = 100 cm^2, other two outlets are circular, $\boldsymbol{S_2}$ = 250 cm^2, $\boldsymbol{S_3}$ = 100 cm^2. These two outlets are located in such a way that there is a perfect contraction during outflow. At the outlet S_1 there is a partial contraction. Determine discharge ***Q*** and water levels in tanks ($\boldsymbol{z_1, z_2, z_3}$), if water level in the first part is in height ***H*** = *3,0 m* above the centre of the last outlet $\boldsymbol{S_3}$. Outflow from this last outlet is free. Tanks of the system can be considered to be sufficiently large.

[Results: 32,9 l.s^{-1};1,35 m; 0,22 m; 1,43 m]

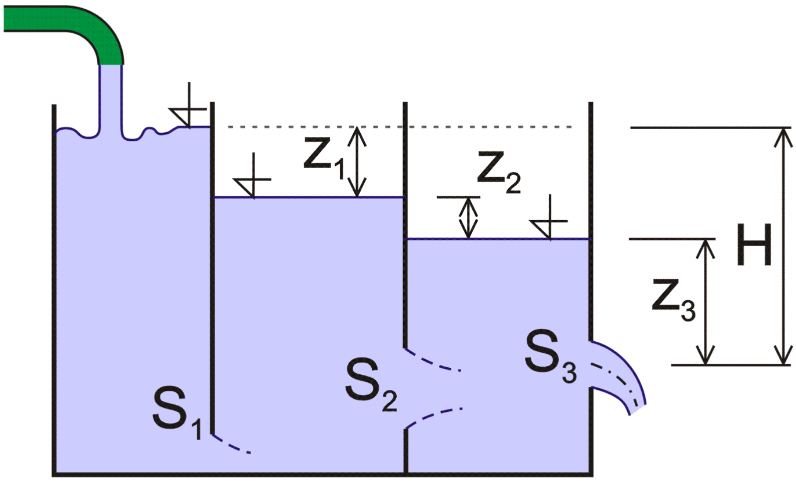

Figure 1

S o l u t i o n

As far as in the first sector there is a stable water level (the position of water level does not change in time), it means that water in the tank neither increases nor decreases. I.e., flow is steady, the constant discharge Q outflows from each outlet.

As the first step of calculation, the outlet in the last wall will preliminary be assumed to be small.

For single outlets, from the equation for discharge $Q = \mu_v \cdot S \cdot \sqrt{2 \cdot g \cdot z}$ it is possible to express:

$$z_i = \frac{Q^2}{2 \cdot g \cdot \mu_{vi}^2 \cdot S_i^2}$$

As $\quad z_1 + z_2 + z_3 = H \quad \Rightarrow$

$$\frac{Q^2}{2 \cdot g}\left(\frac{1}{\mu_{v1}^2 \cdot S_1^2} + \frac{1}{\mu_{v2}^2 \cdot S_2^2} + \frac{1}{\mu_{v3}^2 \cdot S_3^2}\right) = H$$

For circular sharp-edged outlets, the discharge coefficients will be $\mu_{v2} = \mu_{v3} = 0{,}62$, for square outlet, taking into consideration partial contraction, $\mu_{v1} = 0{,}64$

The value of bracket from the previous equation if thus

$$M = \frac{1}{0{,}64^2 \cdot 0{,}01^2} + \frac{1}{0{,}62^2 \cdot 0{,}025^2} + \frac{1}{0{,}62^2 \cdot 0{,}01^2} = 54591$$

$$\Rightarrow \quad Q = \sqrt{\frac{2 \cdot g \cdot H}{M}} = \sqrt{\frac{19{,}62 \cdot 3{,}0}{54591}} = 0{,}0329 \text{ m}^3\text{s}^{-1}$$

$$z_1 = \frac{Q^2}{2 \cdot g \cdot \mu_{v1}^2 \cdot S_1^2} = \frac{0{,}0329^2}{19{,}62 \cdot 0{,}64^2 \cdot 0{,}01^2} = 1{,}35 \text{ m}$$

$$z_2 = \frac{Q^2}{2 \cdot g \cdot \mu_{v2}^2 \cdot S_2^2} = 0{,}22 \text{ m}$$

$$z_3 = \frac{Q^2}{2 \cdot g \cdot \mu_{v3}^2 \cdot S_3^2} = 1{,}43 \text{ m}$$

Second step of calculation: Review of outlet (small or large one?):

Review according to criterion: $(2-3)D_3 < z$

$$D_3 = \sqrt{\frac{4 \cdot S_3}{\pi}} = 0{,}11 \text{ m}$$

Depth of upper edge of outlet bellow the water level

$$z = z_3 - \frac{D_3}{2} = 1{,}43 - \frac{0{,}113}{2} = 1{,}37 \text{ m}$$

$$(2-3) \cdot D_3 = (0{,}22 - 0.33) \text{ m} < z = 1{,}37 \text{ m} \quad \Rightarrow$$

$\Rightarrow$ from hydraulic point of view, the outlet can be considered to be small

$\Rightarrow$ the original assumption was right, it is not necessary to correct the result.

Solved problem 7.2

Water outflows from a pipeline into a pit (inner dimensions of the pit are: ***a*** = 0,4 m, ***b*** = 0,5 m) – see fig. 2. At the bottom of the pit there is a circular outlet with diameter ***D*** = 200 mm. Determine, at which height above the bottom (***h*** = ?) water level will firm up in case that the discharge of 65 $l.s^{-1}$ will flow into the pit. Determine also what discharge should flow into the pit in order to the steady depth of water in the pit was double, in comparison with the depth calculated for the steady flow under the discharge of 65 $l.s^{-1}$.
An average contraction of flow can be considered, with regard to dimensions of ground area of the pit. The outflow coefficient μ_v can be therefore considered to be 0,72.

[Results: 0,32 m; 86,22 $l.s^{-1}$]

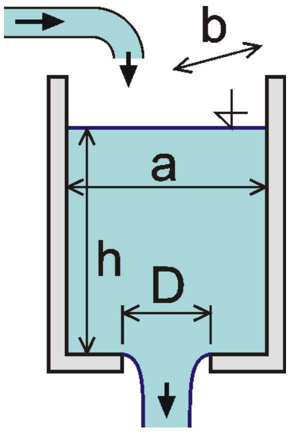

Figure 2

S o l u t i o n

a) Determination of depth *h* for discharge of 65 $l.s^{-1}$:

To solute the problem, the equation of outflow from orifice at the horizontal bottom will be used:

$$Q = \mu_v \cdot S_{orif.} \cdot \sqrt{2 \cdot g \cdot \left(h + \frac{\alpha \cdot v_0^2}{2 \cdot g} + L_c \right)},$$

area of orifice $S_{orif.} = \frac{\pi \cdot D^2}{4} = \frac{\pi \cdot 0{,}2^2}{4} \left[m^2 \right]$

distance of the contracted profile of jet bellow pit bottom

$$L_c \approx 0{,}5 \cdot D = 0{,}5 \cdot 0{,}200 = 0{,}100\ m,$$

velocity of approach $v_0 = \frac{Q}{S_0} = \frac{0{,}065}{0{,}4 \cdot 0{,}5} = 0{,}325\ m.s^{-1} \Rightarrow \frac{\alpha \cdot v_0^2}{2 \cdot g} = 0{,}0054\ m$ (for α = 1)

$$\Rightarrow 0{,}065 = 0{,}72 \cdot \frac{\pi \cdot 0{,}2^2}{4} \cdot \sqrt{2 \cdot 9{,}81 \cdot (h + 0{,}0054 + 0{,}1)} \quad \Rightarrow \quad \underline{h = 0{,}3155\ m}$$

b) Determination of discharge to reach the depth of $2 \cdot h = 0{,}631\, m$:

The equation of outflow from orifice at the horizontal bottom will be used again:

$$Q = \mu_v \cdot S_{orif.} \cdot \sqrt{2 \cdot g \cdot \left(h + \frac{\alpha \cdot v_0^{\ 2}}{2 \cdot g} + L_c \right)}, \qquad S_{orif.} = \frac{\pi \cdot D^2}{4} = \frac{\pi \cdot 0{,}2^2}{4},$$

$$L_c \approx 0{,}5 \cdot D = 0{,}100\ m$$

As discharge is not known yet, the velocity of approach will be neglected at first::

$$Q = 0{,}72 \cdot \frac{\pi \cdot 0{,}2^2}{4} \cdot \sqrt{2 \cdot 9{,}81 \cdot (0{,}631 + 0 + 0{,}1)} = 0{,}085566\ m^3.s^{-1} = 85{,}566\ l.s^{-1}.$$

Now, corrected calculation of velocity of approach and consequent recalculation of discharge can be done:

$$v_0 = \frac{Q}{S_0} = \frac{0{,}085566}{0{,}4 \cdot 0{,}5} = 0{,}428\ m.s^{-1} \quad \Rightarrow \quad \frac{\alpha \cdot v_0^{\ 2}}{2 \cdot g} = 0{,}0094\ m \ \text{(for } \alpha = 1\text{)},$$

$$Q = 0{,}72 \cdot \frac{\pi \cdot 0{,}2^2}{4} \cdot \sqrt{2 \cdot 9{,}81 \cdot (0{,}631 + 0{,}0094 + 0{,}1)} = 0{,}086209\ m^3.s^{-1} = 86{,}209\ l.s^{-1}$$

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As the corrected value of discharge differs about 0,64 ls^{-1} from the previously calculated value, a new correction of velocity of approach will be done. In relation to recalculated velocity of approach also discharge will be recalculated.

$$v_0 = \frac{Q}{S_0} = \frac{0{,}086209}{0{,}4 \cdot 0{,}5} = 0{,}431\, m \cdot s^{-1} \quad \Rightarrow \quad \frac{\alpha \cdot v_0^{\ 2}}{2 \cdot g} = 0{,}0095\ m \ \text{(for } \alpha = 1\text{)},$$

$$Q = 0{,}72 \cdot \frac{\pi \cdot 0{,}2^2}{4} \cdot \sqrt{2 \cdot 9{,}81 \cdot (0{,}631 + 0{,}0095 + 0{,}1)} = 0{,}086216\ m^3.s^{-1} = \underline{86{,}216\ l.s^{-1}}$$

Compared with the previous value, the recalculated discharge has now changed in minimum. Therefore, its value of 86,22 $l.s^{-1}$ can be considered to be the resulting one.

Solved problem 7.3

A fireman keeps a moth of fire hose (***∅D*** = 35 mm) at height 1,3 m above the terrain and is to extinguish a fire in apartment situated 11 m above the terrain level (see fig. 3). Determine the maximum distance from the building in which the fireman has to stand in order to the water jet reaches the extinguished place. Consider the fire water discharge of 19 $l \cdot s^{-1}$.

[Result: 28,4 m]

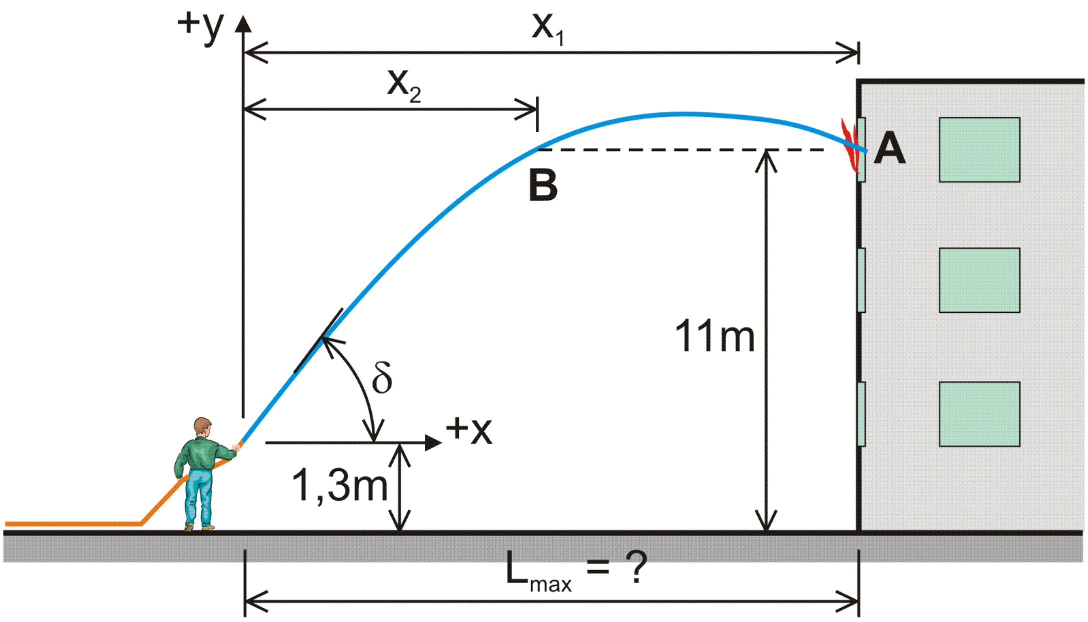

Figure 3

S o l u t i o n

To simplify calculations, the orthogonal coordinate system will be introduced, with the beginning in the outlet profile. Then it is possible to write coordinates of the position of water particle, in dependency on outflow velocity v_0 in which water outflows from the hose, in dependency on time *t* (time of motion of particle from the profile of outflow) and in dependency on angle δ which expresses inclination of hose, taken from horizontal line.

$$x = v_0 \cdot t \cdot \cos\delta, \qquad y = v_0 \cdot t \cdot \sin\delta - \frac{1}{2} \cdot g \cdot t^2 .$$

Expressing time *t* from the first formula and introducing it into the second formula ⟶ the equation will be obtained, describing the water jet trajectory.

$$y = v_0 \cdot \frac{x}{v_0 \cdot \cos\delta} \cdot \sin\delta - \frac{1}{2} \cdot g \cdot \left(\frac{x}{v_0 \cdot \cos\delta} \right)^2 ,$$

mathematical arrangement: $$y = x \cdot tg\delta - \frac{1}{2} \cdot g \cdot \frac{x^2}{{v_0}^2 \cdot \cos^2\delta} .$$

In this case the coordinate $y = 11 - 1{,}3 = 9{,}7\ \mathrm{m}$, and the coordinate x has to be determined.

Using the continuity equation it is possible to express the outflow velocity:

$$v_0 = \frac{Q}{S_0} = \frac{Q}{\frac{\pi \cdot D^2}{4}} = \frac{19 \cdot 10^{-3}}{\frac{\pi \cdot 0{,}035^2}{4}} = 19{,}748\ \mathrm{m \cdot s^{-1}}$$

Introducing this velocity into equation, expressing the jet shape, and by mathematical arrangement:

$$\frac{1}{2} \cdot g \cdot \frac{1}{19{,}748^2 \cdot \cos^2\delta} \cdot x^2 - x \cdot \mathrm{tg}\delta + 9{,}7 = 0.$$

Quadratic equation roots:

$$x_1 = \frac{\mathrm{tg}\delta + \sqrt{\mathrm{tg}^2 - 4 \cdot \frac{1}{2} \cdot g \cdot \frac{1}{19{,}748^2 \cdot \cos^2\delta} \cdot 9{,}7}}{2 \cdot \frac{1}{2} \cdot g \cdot \frac{1}{19{,}748^2 \cdot \cos^2\delta}}$$

$$x_2 = \frac{\mathrm{tg}\delta - \sqrt{\mathrm{tg}^2 - 4 \cdot \frac{1}{2} \cdot g \cdot \frac{1}{19{,}748^2 \cdot \cos^2\delta} \cdot 9{,}7}}{2 \cdot \frac{1}{2} \cdot g \cdot \frac{1}{19{,}748^2 \cdot \cos^2\delta}}$$

Demanded solution (maximum distance *L*) is represented by maximum value of x according to the equation root x_1 (see figure). This maximum can be solved or analytically (expressing the derivation *x* about δ and its solution for value equal to 0), or by solving the equation for expressing the root x_1, with several choice of angle δ, and its sequential refinement (such an angle δ is searched in which x_1 issues as maximum value). In the following table there are figured out the results of x_1, but, for the complete imagination, also values of root x_2 representing point *B* at the figure:

δ [°]	x_1 [m]	x_2 [m]
45	22,957	16,797
50	27,609	11,542
54,4	**28,446**	**9,187**
55	28,433	8,924
60	27,389	7,040

Conclusion:

Maximum distance of the fireman from the building is (rounded in whole dm) 28,4 m.

Solved problem 7.4

Determine the arcing distance of sprinkling installation. Overpressure in chamber A, from which water outflows by nozzles B, is $p_{pA} = 80$ kPa. Velocity coefficient of nozzles consider to be $\varphi = 0{,}90$. Calculate the arcing distance in variants, for angle α (inclination of nozzle axis from horizontal line) being 35°, 45°and 55°.

[Results: 12,14 m; 13,21 m; 12,41 m]

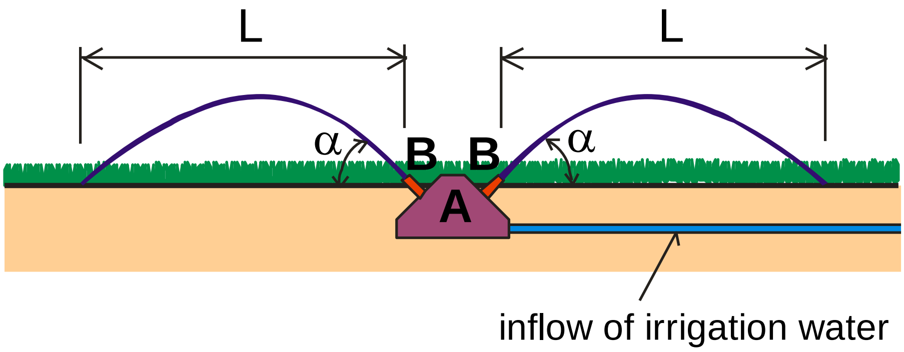

Figure 4

S o l u t i o n

From the equation of the jet trajectory, arcing distance will be calculated:

$$L = 2 \cdot v_0^{\;2} \cdot \frac{1}{g} \cdot \sin\alpha \cdot \cos\alpha = \frac{v_0^{\;2} \cdot \sin(2\alpha)}{g},$$

velocity at the end of nozzle: $v_0 = \varphi \cdot \sqrt{2 \cdot g \cdot H}$,

pressure head in chamber A: $H = \frac{p_{pA}}{\rho \cdot g} = \frac{80 \cdot 10^3}{1000 \cdot 9{,}81} = 8{,}155\ m\, w.\, col.$

$$\Rightarrow v_0 = 0{,}90 \cdot \sqrt{2 \cdot 9{,}81 \cdot 8{,}155} = 11{,}384\ m.s^{-1} \Rightarrow$$

$$L = \frac{11{,}384^2 \cdot \sin(2\alpha)}{9{,}81} = 13{,}211 \cdot \sin(2\alpha)$$

for $\alpha = 35°$ ⇒ <u>L = 12,414 m</u>,
$\alpha = 45°$ ⇒ <u>L = 13,211 m</u>,
$\alpha = 55°$ ⇒ <u>L = 12,414 m</u> .