Solved problems – 2nd exercise

Problem 2.1.

Determine the magnitude and point of action of force **F** which loads an inclined forehead of the cistern containing oil fuel (see figure 1). Diameter of the cistern **D = 2 m**, **α = 60°**, Above the level there is the overpressure **p_p = 30 kPa**. The density of the oil fuel can be considered by the value **ρ_o = 900 kg·m^{-3}**.

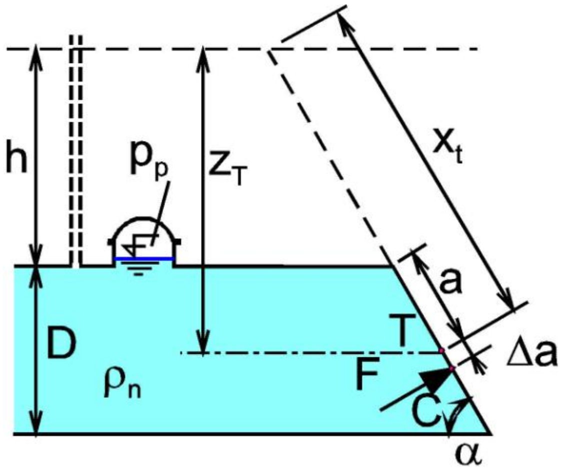

Figure 1

S o l u t i o n:

1) Force $F = \rho_o \cdot g \cdot S \cdot z_T$; $S = \pi \cdot a \cdot b$; $a = \frac{D}{2\sin\alpha} = 1{,}15$ m ; $b = \frac{D}{2} = 1{,}0$ m

The centre of the forehead is situated in the depth D/2. However, as there is the overpressure above the level, to calculate z_T it is necessary to enlarge the position of free level by the overpressure head $h = \frac{p_p}{\rho_o \cdot g}$, that is to determine position of pressure line (fictional free level)

$$h = \frac{p_p}{\rho_o \cdot g} = \frac{30000}{900 \cdot 9{,}81} = 3{,}4 \text{ m}; \quad z_T = h + D/2 \quad \Rightarrow$$

$$\Rightarrow \quad F = \rho_n \cdot g \cdot \pi \cdot a \cdot b \cdot \left(h + \frac{D}{2}\right) = 140{,}35 \text{ kN}$$

2) Point of action of hydrostatic force:

$$x_T = \frac{z_T}{\sin\alpha} = \frac{h + \frac{D}{2}}{\sin\alpha} = \frac{3{,}4 + 1}{\sin 60^\circ} = 5{,}08 \text{ m}; \quad \Delta a = \frac{I_o}{S \cdot x_T} = \frac{\pi \cdot a^3 \cdot b}{4 \cdot \pi \cdot a \cdot b \cdot x_T} = 0{,}066 \text{ m}$$

$$x_C = x_T + \Delta a = 5.08 + 0.066 = \underline{5.146 \text{m}}$$

Problem 2.2.

Calculate the magnitude of force **F**, which is needed to jack up the cover which obstructs an outlet of the reservoir with water (see figure 1). Depth of water in the reservoir **h = 1,0 m**, **a = 0,5 m**, **α = 60º**, weight of the cover **G = 1,2 kN**. Calculate the force considering **1 m'** of the width of the cover. Friction can be neglected.

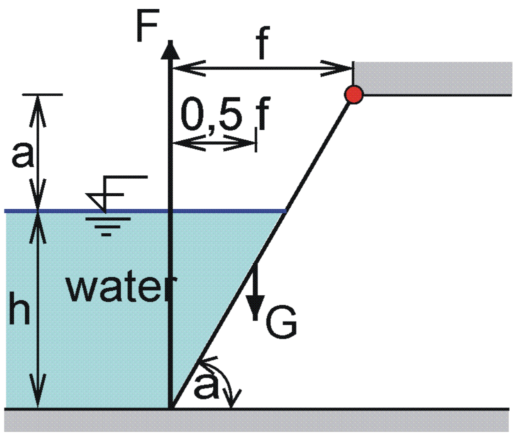

Figure 2

Solution :

h/sin·α = 1,155 ; a/sinα = 0,577 m ; L = 1,55 + 0,577 = 1,732 m.

f = L·cosα = 1,732·cos 60° = 0,866 m.

Hydrostatic force → see fig. 3:

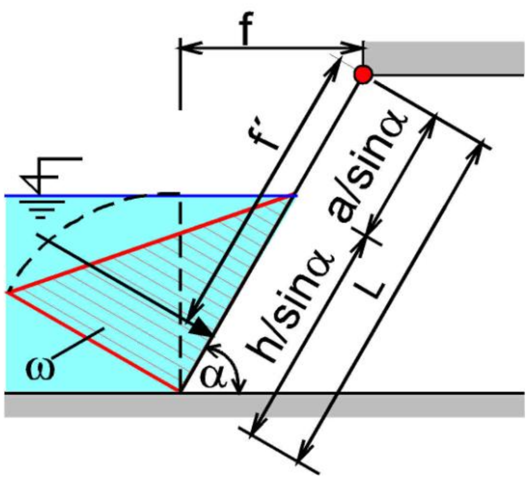

Figure 3

$$F' = \rho \cdot g \cdot \varpi = \rho \cdot g \cdot \frac{h^2}{\sin\alpha} \cdot \frac{1}{2} = 1000 \cdot 9{,}81 \cdot \frac{1}{2} \cdot \frac{1}{\sin 60^\circ} = 5664 \text{ N}$$

Necessary force will be calculated from the condition of moment equilibrium, calculated to the axis of turning of the cover

$$f' = \frac{a}{\sin\alpha} + \frac{2}{3} \cdot \frac{h}{\sin\alpha} = 0{,}577 + \frac{2}{3} \cdot 1{,}155 = 1{,}347 \text{ m}$$

$$F' \cdot f' + G \cdot \frac{f}{2} = F \cdot f$$

$$F = \frac{F' \cdot f' + G \cdot \frac{f}{2}}{f} = \frac{5664 \cdot 1{,}347 + 1200 \cdot 0{,}433}{0{,}866} = \underline{9410\,\text{N}}$$

Problem 2.3.

Calculate a magnitude of hydrostatic force **F**, which loads **1 m´** of concrete gravity dam (see fig. 4) and propose a slope of the outer face of the dam in such a way that the stability of the dam agains shifting in footing bottom will be ensured. Coefficient of safety **μ = 1,25**, friction coefficient **φ = 0,7,** density of concrete **ρ_c = 2400 kg·m^{-3}**.

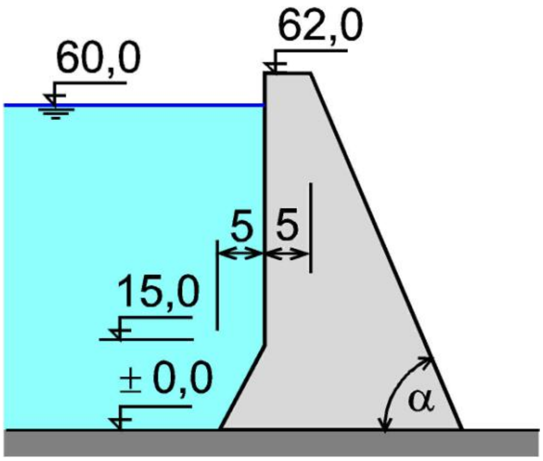

Figure 4

S o l u t I o n:

Components of the hydrostatic force can be deternined from pressure diagrams – see fig. 5

$$F_x = \rho \cdot g \cdot \omega_X = 1000 \cdot 9{,}81 \cdot 0{,}5 \cdot 60^2 = 17\ 658 \text{ kN}$$

$$F_z = \rho \cdot g \cdot \omega_Z = 1000 \cdot 9{,}81 \cdot 5 \cdot 0{,}5\ (45 + 60) = 2\ 575{,}125 \text{ kN}$$

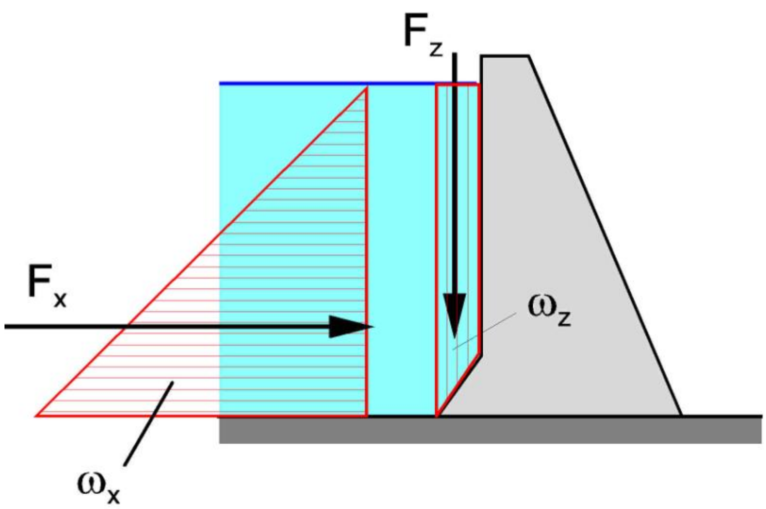

Figure 5

Weight of the dam:

$G = \rho_c \cdot g\ (5 \cdot 62 + 0{,}5 \cdot 5 \cdot 15 + 0{,}5 \cdot 62^2 \cdot \mathrm{cotg}\alpha) = 8\ 181{,}54 + 45\ 251{,}57 \cdot \mathrm{cotg}\alpha$ kN

$T = \varphi\ (F_Z + G) = 0{,}7 \cdot (2\ 575{,}125 + 8\ 181{,}54 + 45\ 251{,}57 \cdot \mathrm{cotg}\alpha =$

$= 7\ 529{,}67 + 31\ 676{,}1 \cdot \mathrm{cotg}\alpha$ kN

$\mu\ F_x = 1{,}25 \cdot 17\ 658{,}0 = 22\ 072{,}5$ kN

$T \geq \mu\ F_X$

$7\ 529{,}67 + 31\ 676{,}1 \cdot \mathrm{cotg}\ \alpha = 22\ 072{,}5$

from which

$$\mathrm{tg}\alpha = \frac{31676{,}1}{14542{,}83} = 2{,}17812\ ;\quad \alpha = 65^\circ 20'$$

Problem 2.4.

A rotary tank is hanging on the cable. At figure 6 a vertical section through the tank can be seen.

Determine:

a) magnitude of hydrostatic force acting on the tank bottom, F_{bottom}

b) magnitude of total hydrostatic force acting on the tank, F

c) magnitude of force $F_{screw\text{-}bottom}$ acting axially on one screw from the total number of 24. These screws connect the tank bottom with the vertical part of the tank.

d) magnitude of force $F_{screw\text{-}vertical}$ acting axially on one screw from the total number of 24. These screws connect the upper conical part of the tank with its vertical cylindrical part.

The tank is filled with water. Its dimensions are

h_1 = 0,4 m, h_2 = 0,4 m, h_3 = 1,0 m, D_1 = 0,25 m, D_2 = 1,0 m.

The upper part of the tank weights 33 kg, the middle cylindrical part weights 54 kg and bottom weights 16 kg.

(Result: 13,87 kN; 9,25 kN; 584,4 kN; 178,9kN)

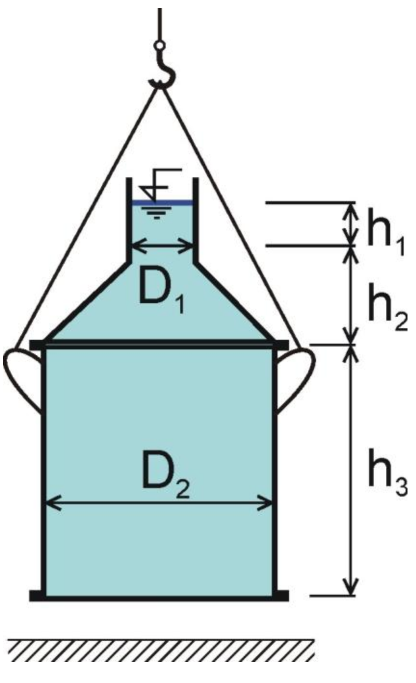

Figure 6

S o l u t i o n:

a) Hydrostatic force acting on bottom:

$$F_{bottom} = \rho \cdot g \cdot \frac{\pi \cdot D_2^2}{4} \cdot (h_1 + h_2 + h_3) = 1000 \cdot 9{,}81 \cdot \frac{\pi}{4} \cdot (0{,}4 + 0{,}4 + 1{,}0) = 13\ 868{,}6\ N$$

b) The total hydrostatic force acting on the tank.

Hydrostatic force acting on upper part of the tank:

Hydrostatic force acts in the direction perpendicular to the loaded area. That is, that forces $\mathbf{F_{AB}}$ in every axis section form a pair of counteracting forces ⇒ their total action on the tank is zero – see fig. 7

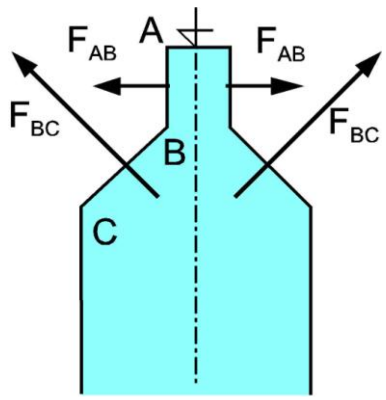

Figure 7

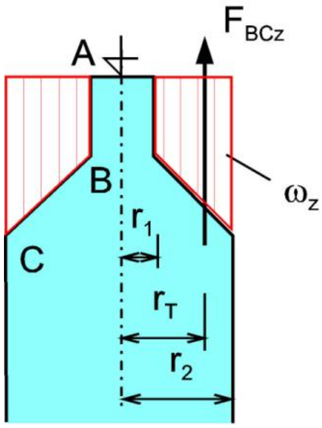

Figure 8

In axis sections it is possible to distribute forces F_{BC} into horizontal (F_{BCx}) and vertical (F_{BCz}) components. The result of acting of horizontal components is again zero.

The resulting hydrostatic force acting on upper part of the tank is therefore given by the weight of pressure body of the vertical component of force, $F = \rho \cdot g \cdot V_z$ (see fig. 8). The pressure body volume (rotary body) is $V_z = 2 \cdot \pi \cdot r_T \cdot \omega_z$.

$$\omega_z = (r_2 - r_1) \cdot h_1 + 0{,}5 \cdot (r_2 - r_1)\ h_2 = (0{,}5 - 0{,}125)\ 0{,}4 + 0{,}5 \cdot (0{,}5 - 0{,}125)\ 0{,}4 = 0{,}225\ m^2$$

Magnitude of $\mathbf{r_T}$ can be determined by calculation of centre of gravity of compound diagram:

$$r_T = r_1 + \frac{r_{T\ obdélník} \cdot \omega_{z\ obdélník} + r_{T\ trojúhelník} \cdot \omega_{z\ trojúhelník}}{\omega_z} =$$

$$= r_1 + \frac{\frac{r_2 - r_1}{2} \cdot (r_2 - r_1) \cdot h_1 + \frac{2}{3}(r_2 - r_1) \cdot 0{,}5 \cdot (r_2 - r_1) \cdot h_2}{(r_2 - r_1) \cdot h_1 + 0{,}5 \cdot (r_2 - r_1) \cdot h_2} =$$

$$= 0{,}125 + \frac{\frac{0{,}5 - 0{,}125}{2} \cdot (0{,}5 - 0{,}125) \cdot 0{,}4 + \frac{2}{3}(0{,}5 - 0{,}125) \cdot 0{,}5 \cdot (0{,}5 - 0{,}125) \cdot 0{,}4}{(0{,}5 - 0{,}125) \cdot 0{,}4 + 0{,}5 \cdot (0{,}5 - 0{,}125) \cdot 0{,}4} =$$

$$= 0{,}333 m$$

$V_z = 2 \cdot \pi \cdot r_T \cdot \omega_z = 2 \cdot \pi \cdot 0{,}333 \cdot 0{,}225 = 0{,}471\ m^3$

$\uparrow F_{upper\ part} = \rho \cdot g \cdot V_z = 1000.9{,}81.0{,}471 = 4\ 620{,}5\ N$

Hydrostatic force acting on vertical cylindrical part of the tank

It is zero – from the same reason as the force F_{AB} acting on cylindrical part of the upper part of the tank.

Resulting hydrostatic force acting on the tank is vertical, given by the difference of hydrostatic force acting down on the tank bottom and hydrostatic vertical force but acting up on the conical shape of the upper part of the tank.

$F = \downarrow F_{bottom} - F_{upper\ part} = 13\ 868{,}6 - 4620{,}51 = 9248{,}09\ N$

c) Magnitude of the power acting axially on one screw from the total number of 24 ($F_{screw\text{-}bottom}$ = ?) – connection of tank bottom and vertical part of the tank.

Screws, connecting the tank bottom and tank vertical part will have to transfer powers acting on the tank bottom, i.e. hydrostatic force acting on the tank bottom, and own gravity of the bottom. One screw will transfer 1/24 of the total force:

$$F_{screw\text{-}bottom} = \frac{1}{24}(F_{bottom} + G_{bottom}) = \frac{1}{24}(13868{,}6 + 16 \cdot 9{,}81) = \underline{584{,}4\ N}$$

c) Magnitude of the power acting axially on one screw from the total number of 24 ($F_{screw\text{-}vertica}$ = ?) – connection of upper conical part and vertical cylindrical part of the tank.

Screws, connecting upper part and middle cylindrical part of the tank transfer the resulting force acting at the vertical direction up to the conical part of the tank. This force equals to 1/24 of the difference of resulting hydrostatic force acting up on upper conical shape of the tank, and own gravity of this part of the tank:

$$F_{screw\text{-}vertical} = \frac{1}{24}\left(F_{upper\ part} - G_{upper\ part}\right) = \frac{1}{24}\left(4618{,}2 - 33 \cdot 9{,}81\right) = \underline{178{,}9\ N}$$